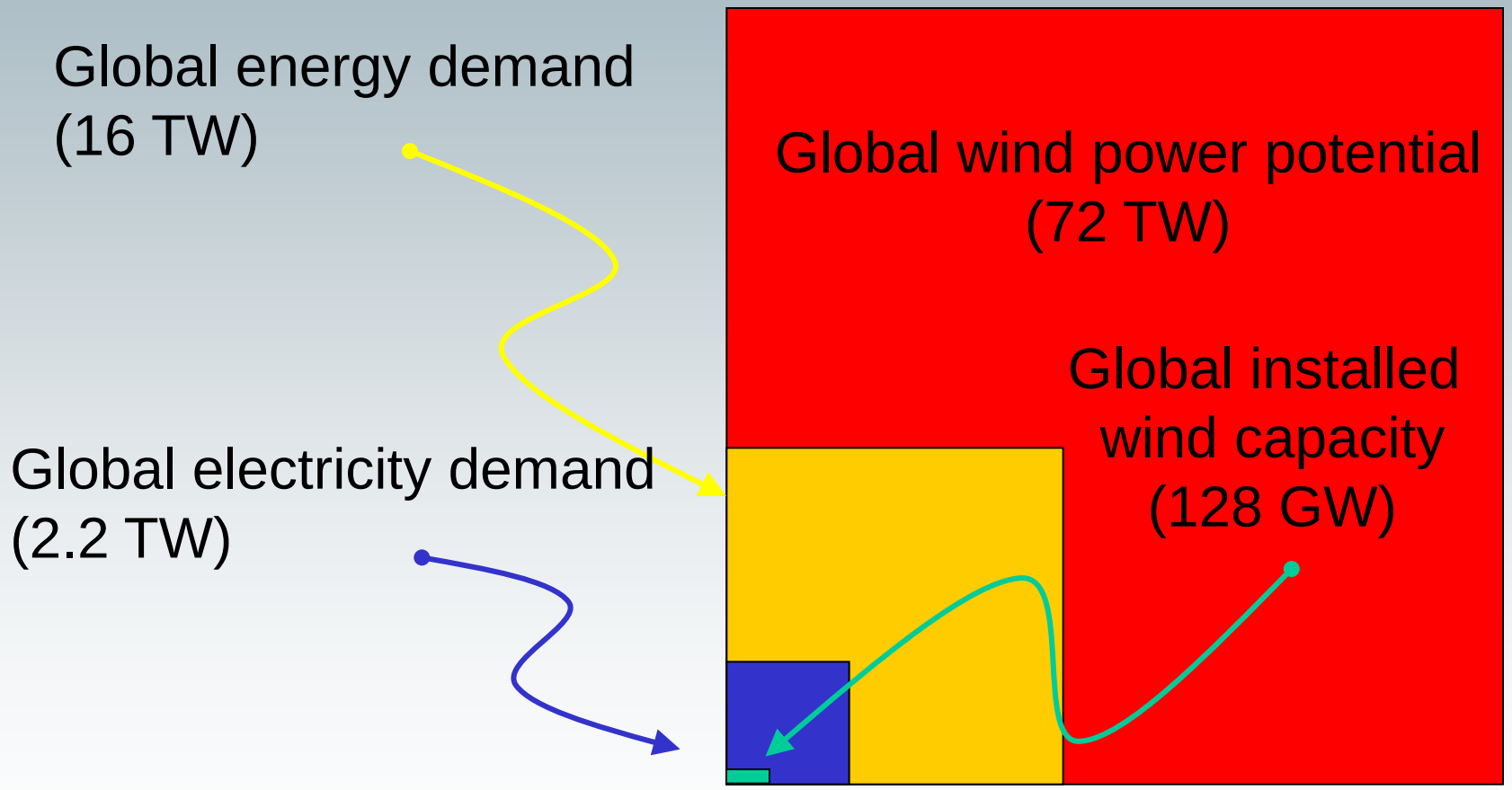
Optimal Power Flow

Steven Low

Caltech

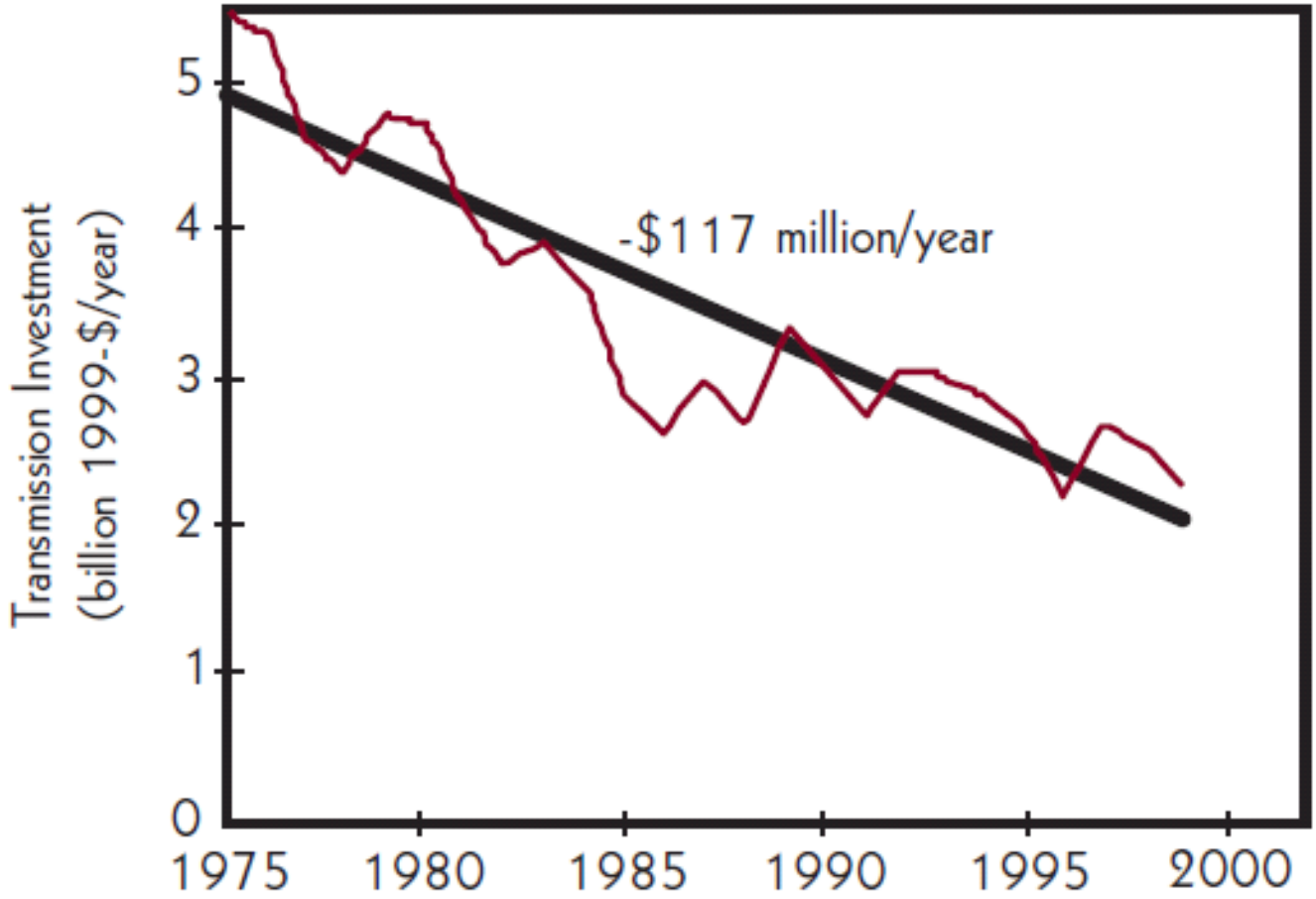
Consistent result from all methods: global wind potential is ~100 TW



Source: Cristinea Archer, 2010

U.S. Transmission Investments

Annual investment in transmission facilities has been declining since 1975.



Source: U.S. DOE National Transmission Grid Study May 2002

Challenges

One of the largest man made systems

- nonlinear, distributed, feedback, delayed, stochastic

Technical challenges

1. Uncertainty in demand and supply
2. Wide range of timescales
3. SoS architecture and algorithms

Control & optimization of SG

- Holistic approach
  - Real-time control, storage, reliability (as a resource, not worst-case mentality), demand response (exploit deferrable loads)
- Distributed architecture & algorithms
  - Internet: architecture is the single most important element underlying its success
  - Widely recognized, rarely formalized
- Dynamic control & optimization
  - It will be possible to control 1,000x faster
  - The bottleneck problem is models & algorithms for analysis and control

Example theory advances

- Characterize OPF subproblems that are polynomial-time solvable

- Propose a new approach to solve OPF
  - Efficient and effective (global optimal)
  - Through solving a SDP
  - A sufficient condition under which optimal OPF solution can be recovered

- Works for all IEEE benchmark systems!
  - All satisfy the condition
  - 14, 30, 57, 118, 300 buses

Dual OPF : SDP

$$\max_{x \geq 0, r} \; h(x, r)$$

Linear function

subject to

$$\sum_{k=1}^{n} \left(\boldsymbol{\lambda}_k \mathbf{Y}_k + \bar{\boldsymbol{\lambda}}_k \bar{\mathbf{Y}}_k + \boldsymbol{\mu}_k M_k \right) \succeq 0$$

$$\begin{bmatrix} 1 & r_{l1} \\ r_{l1} & r_{l2} \end{bmatrix} \succeq 0, \quad l = 1, 2, ..., m$$

$$x := (\lambda_k^{\min}, \lambda_k^{\max}, \bar{\lambda}_k^{\min}, \bar{\lambda}_k^{\max}, \mu_k^{\min}, \mu_k^{\max}, \; k = 1, ..., n)$$

Our proposal

- Solve convex dual OPF for (x^{opt}, r^{opt})
- Compute $[U_1^T \quad U_1^T]^T$ in the null space of

$$A^{\text{opt}} := \sum_{k=1}^{n} \left(\lambda_k^{\text{opt}} \mathbf{Y}_k + \bar{\lambda}_k^{\text{opt}} \bar{\mathbf{Y}}_k + \mu_k^{\text{opt}} M_k \right)$$

- Compute a primal solution

$$V^{\text{opt}} = (\zeta_1 + \zeta_2 \mathrm{i})(U_1 + U_2 \mathrm{i})$$

- If it is primal feasible, it is globally optimal

Sufficient condition

<u>Theorem</u>

Under a condition,

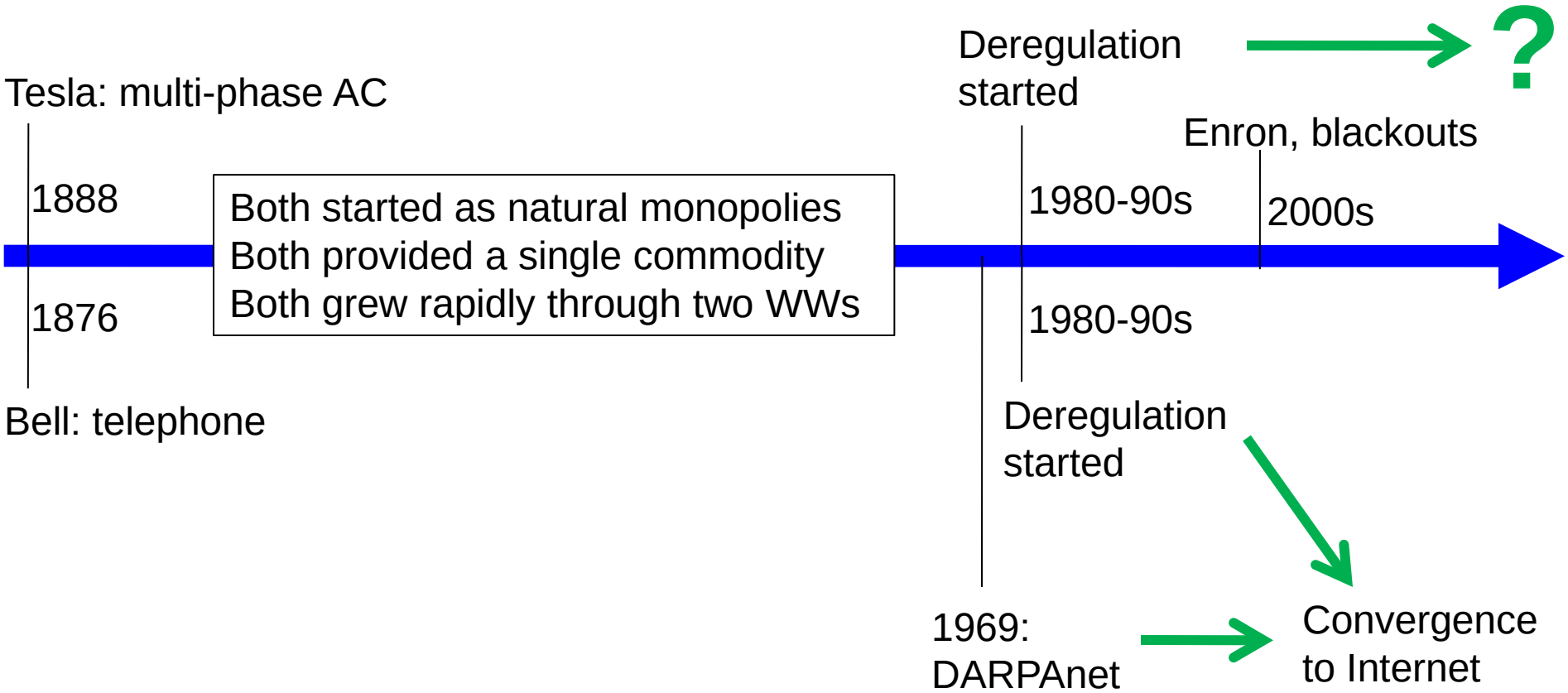
- The duality gap is zero
- V^{opt} is globally optimal

<u>Implication</u>:

- Allows <u>faster</u> static control, towards dynamic control
- Recent advances can help, but much work remains

SoS architecture

Power network will go through similar **architectural transformation** in the next couple decades that phone network has gone through



SoS architecture

... to become more interactive, more distributed, more open, more autonomous, and with greater user participation

What is an architecture theory to help guide the transformation?

... while maintaining security & reliability